Equivalent Circuit Method for Analysis of Modified Jerusalem Cross FSS

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***Abstract*—In this paper, an Equivalent Circuit Method (ECM) is used for analyzing the Modified Jerusalem Cross Frequency Selective Surfaces (MJC-FSS). The method consists in estimating the frequency response of an FSS by calculating the inductive reactances and capacitive susceptibilities related to the structure. To validate the results, insertion loss (S21) were performed using a 3D full-wave electromagnetic simulation software.**

Keywords*—*frequency selective surface; multiband FSS; Jerusalem Cross FSS

I. INTRODUCTION

The response in the electromagnetic spectrum of a Frequency Selective Surface (FSS) is usually estimated by commercial computer programs of electromagnetic simulation by the use of full wave analysis. These programs perform a very complete verification of the characteristics and operation of the FSSs, however they require enormous computational effort, a simple problem can take a few hours to be solved.

To obtain immediately the electromagnetic properties of an FSS, a useful Equivalent Circuit Method (ECM) consists in relating passive discrete filters to this surface. The ECM is also an alternative to reduce the computational effort and consequently the simulation time. The process of defining the discrete components of equivalent circuits related to periodic structures is initially given by the representation of an infinite array of parallel conducting strips [1].

II. MODIFIED JERUSALEM CROSS EQUIVALENT CIRCUIT

The ECM is used to analyze a modified Jerusalem Cross FSS was chosen (MJC-FSS) [2]. This Structure consists of the replication of JC geometry at different scales to provide a multiband structure without changing the area occupied by the primary structure. Fig. 1 shows the basic structure FSS-JC, the three iterations for the FSS-JC and its respective discrete circuits. The discrete circuit models were based on the Leonard and Cofer's work [6], where relate the FSS-JC with a resonant circuit. The technique used for the calculation of discrete components of the circuit was proposed in [3], which calculates the inductive reactance (X_L) and the capacitive susceptance (B_C) of conductive strips, as follows

$$X_L = F(p,w,\lambda,\phi) = \frac{p\cos\phi}{\lambda}\left\{\ln\left[cosec\left(\frac{\pi w}{2p}\right)\right] + G(p,w,\lambda,\phi)\right\} \quad (1)$$

$$\frac{B_C}{Z_0} = 4F(p,w,\lambda,\phi) \quad (2)$$

Where p is the periodicity, w is the strip width, λ is the wavelength and ϕ is the incident angle of the plane wave on the strips. The function G (p, w, λ, φ) and its parameters are defined below.

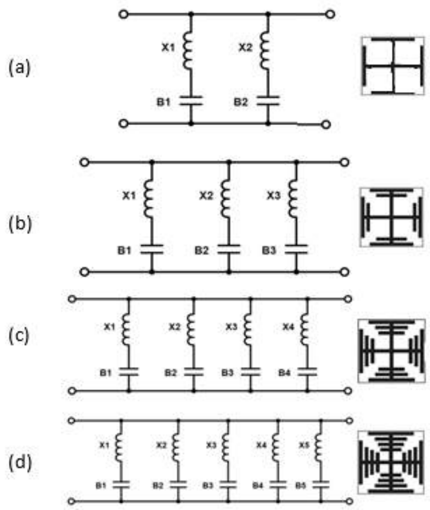

Fig. 1. Equivalent Circuits for four Modified Jerusalem Cross FSS Iterations: (a) Iteration 0 (FSS-JC); (b) Iteration 1; (c) Iteration 2 e (b) Iteration 3.

$$G(p,w,\lambda,\phi) = \frac{0{,}5(1-\beta^2)^2\left[\left(1-\frac{\beta^2}{4}\right)(C_+ + C_-) + 4C_+C_-\beta^2\right]}{\left(1-\frac{\beta^2}{4}\right)+\left(1+\frac{\beta^2}{4}-\frac{\beta^4}{8}\right)(C_+ + C_-) + 2C_+C_-\beta^6} \quad (3)$$

$$C_{\pm} = \frac{1}{\sqrt{1 \pm \frac{2p\,sen\emptyset}{\lambda} - \left(\frac{p\cos\emptyset}{\lambda}\right)^2}} - 1 \quad (4)$$

$$\beta = sen\left(\frac{\pi w}{2p}\right) \quad (5)$$

From (1) and (2), the reactance and susceptance values of the equivalent circuit are related to a freestanding (without dielectric substrate) FSS [3]. An advantage of this method is that the electrical parameters of the circuit (inductance and capacitance) are direct function of the physical parameters (dimensions) of the FSS. The equivalent circuit for FSS have already been obtained from the theory of conducting strips, such as: square loop [4], double square loop [5], square loop with grid [6], Jerusalem Cross [7], etc.

The reactance X_{L1} is calculated from (1) representing the inductance of the central dipole, the susceptibility B_{C1} represents the capacitance generated by dipoles spaced apart from g.

$$B_{c1} = \frac{4l_0}{p}F(p,g,\lambda,\phi) + \frac{4(2w'+g)}{p}F(p,p-l_0,\lambda,\phi) \quad (6)$$

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Where l_0 is the length of the central dipole, *w'* is the width related to the dipoles at the ends of the FSS-JC.

The X_{L2}, which represents the inductance of the dipole at the ends, is determined from (7), while that the B_{C2} will not be calculated using direct analysis, as described above, but from resonant frequency f_2 relating to $\lambda / 2$ [8]. The capacitance C_2 is derived from the equation of the resonant circuit in series, thus, one can estimate the value of B_{C2} [9].

$$X_{L2} = \frac{l_1}{p} F(p, 2w, \lambda_2, \phi) \tag{7}$$

$$B_{C2} = \frac{1}{X_{L2}(2\pi f_2)^2} \tag{8}$$

Where l_1 is the length of the dipoles at the ends of the FSS-JC and λ_2 is the wavelength related to the second resonance frequency.

The modeling technique for the new dipoles distributed in the FSS-JCM follows the same principle of the dipoles distributed at the ends of the FSS-JC. As new resonances are expected, each new dipole will be associated with a series LC circuit, whose inductive reactance and capacitive susceptance will be calculated from (7) and (8), as shown in Fig.1.

Fig. 2 shows four unit cells of the traditional FSS-JC, and Fig. 3 shows the unit cells for the four iterations of the FSS-MJC that will be modeled and analyzed in this paper.

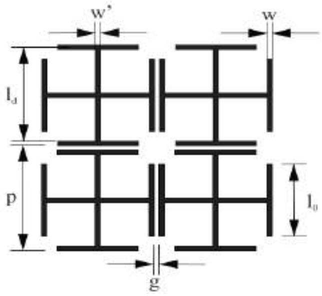


Fig. 2. Four unit cells of the traditional Jerusalem Cross FSS

The FSS studied here were designed according to the parameters presented in Tables I and II.

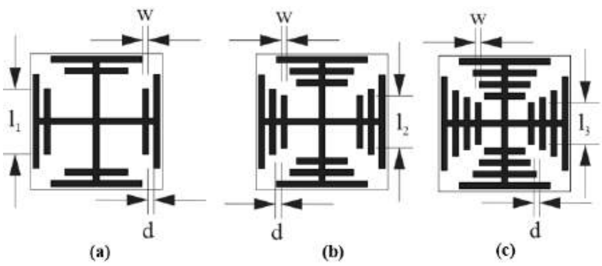


Fig. 3. Unit cell of the Modified Jerusalem Cross FSS: (a) Iteration 1 (1 dipole added); (b) Iteration 2 (2 dipole added); (c) Iteration 3 (3 dipole added).

TABLE I. LENGTH OF THE DIPOLES

Parameter	l_d	l_0	l_1	l_2	l_3
Length (mm)	22.50	16.10	11.20	8.96	7.16

TABLE II. GENERAL PARAMETERS OF THE FSS

Parameter	*p*	*d*	*g*	*W*	*W'*
Length (mm)	24.50	1.00	1.00	1.00	1.00

III. ANALYSIS OF THE MODIFIED JERUSALEM CROSS FSS USING THE EQUIVALENT CIRCUIT METHOD (ECM)

A MATLAB® routine was developed based on [5], which calculates and stores the transmission coefficient from the values of the susceptances and admittances to a freestanding FSS.

A correction for the resonance frequency is necessary taking into account the effect of the dielectric layers [4,10].

$$f_r = \frac{f}{\sqrt{\varepsilon_{eff}}} \tag{9}$$

The effective dielectric constant is calculated as shown below.

$$\varepsilon_{eff} = (\varepsilon_r + 1)/2 \tag{10}$$

For the calculation of the transmission coefficient, it is necessary to calculate the total circuit admittance, which consists of the sum of the individual admittances of each LC series circuit, as follows

$$T = \frac{1}{\sqrt{1 + 0.25(Y_{TOTAL})^2}} \tag{11}$$

$$Y_{TOTAL} = Y_{LC1} + Y_{LC2} + Y_{LC3} + \cdots + Y_{LCn} \tag{12}$$

$$Y_{LC} = \frac{1}{X_L - \frac{1}{B_C}} \tag{13}$$

The magnitude in dB of the transmission coefficient is

$$T_{dB} = 20 \log_{10} |T| \tag{14}$$

The values for the reactances and susceptances are expressed in Tables III and IV, respectively.

TABLE III. CALCULATED VALUES FOR THE REACTANCES

Reactances	X_{L1}	X_{L2}	X_{L3}	X_{L4}	X_{L5}
Ω	4.19	1.20	0.53	0.53	0.52

TABLE IV. CALCULATED VALUES FOR THE SUSCEPTANCES

Susceptances	B_{C1}	B_{C2}	B_{C3}	B_{C4}	B_{C5}
Ω^{-1}	6.77	5.88	6.61	4.24	2.71

IV. RESULTS

A 3D full-wave electromagnetic simulation software was used to validate the results of the Equivalent Circuit Method (ECM). First, the FSS-JCM was simulated with the ideal configuration (freestanding), as shown in Fig. 4. Table V shows the resonant frequencies for the two methods and the percentage difference between them. Analyzing the percentage differences, the MCE model for freestanding FSS is very close to the simulations using the 3D full-wave EM software. It presented a maximum difference of 6.13% relating to the first resonance.

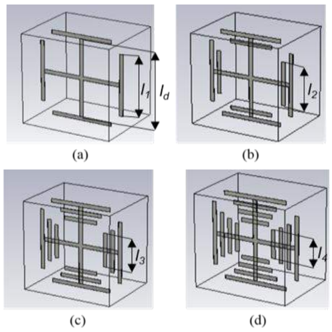
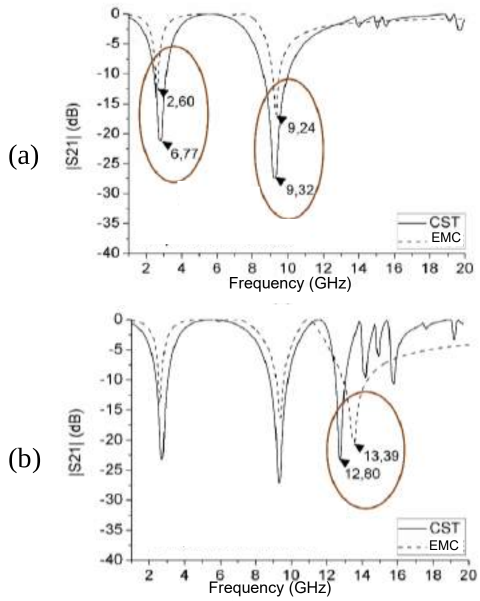


Fig. 4. 3D view of the freestanding FSS-JCM in the 3D Full-Wave Software: a) Iteration 0; (b) Iteration 1; (c) Iteration 2; (d) Iteration 3.

TABLE V. RESONAT FREQUENCIES FOR THE FREESTANDING FSS

	Software (GHz)	MCE (GHz)	Software /MCE (%)
f_1	2.77	2.60	6.13
f_2	9.24	9.32	0.86
f_3	12.80	13.39	4.61
f_4	16.83	16.74	0.84
f_5	20.75	20.95	0.96

Figure 5 presents the comparison between the results for all iterations, between the model simulated in the 3D Full-Wave EM Software and the MCE in the freestanding configuration both. The markings in the figures represent the frequencies analyzed in each iteration.



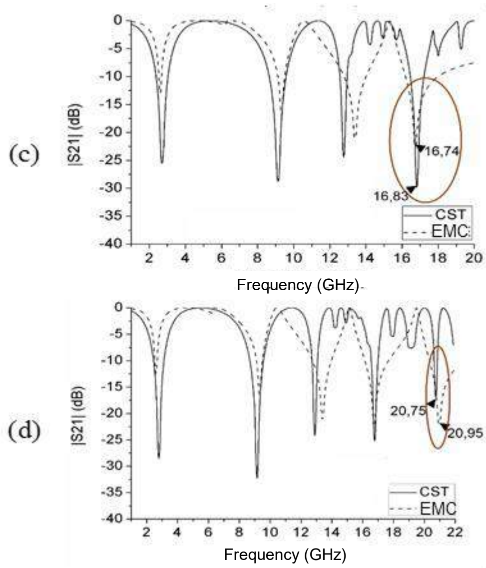


Fig. 5. S21 Comparison of the freestanding FSS-JCM iterations using the Software and ECM: (a) Iteration 0; (b) Iteration 1; (c) Iteration 2; (d) Iteration 3.

Even the freestanding model is not feasible, the analysis described above is of great importance because it is the basis for the modeling of equivalent circuits with the dielectric layer. Therefore, the next step is to validate the MCE with dielectric effect.

A dielectric substrate FR-4 (h = 1 mm, εr = 4.4, tanδ = 0.02) was added to the freestanding structures. As shown in Fig.6.

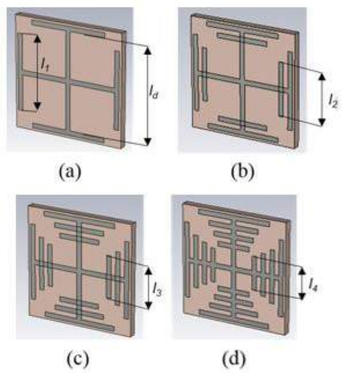


Fig. 6. 3D view of the FSS-JCM with dielectric substrate: a) Iteration 0; (b) Iteration 1; (c) Iteration 2; (d) Iteration 3.

Table VI presents the resonant frequencies for the simulated prototypes in the Software and modeled by the ECM, and the percentage difference between them. It can be observed that the results generated by the MCE are very close to the results simulated in the Software. The maximum difference was 3.71% for the second resonance frequency.

Fig 7 shows the comparison between the results for the four iterations of ECM and Software. The orange marking represents the frequencies analyzed for each iteration.

TABELA VI. RESONAT FREQUENCIES FOR THE FSS WITH DIELECTRIC SUBSTRATE

	Software (GHz)	MCE (GHz)	Software /MCE (%)
f_1	1.78	1.73	2.80
f_2	6.42	6.21	3.27
f_3	8.69	8.90	2.41
f_4	10.94	11.16	2.01
f_5	12.98	12.91	0.53

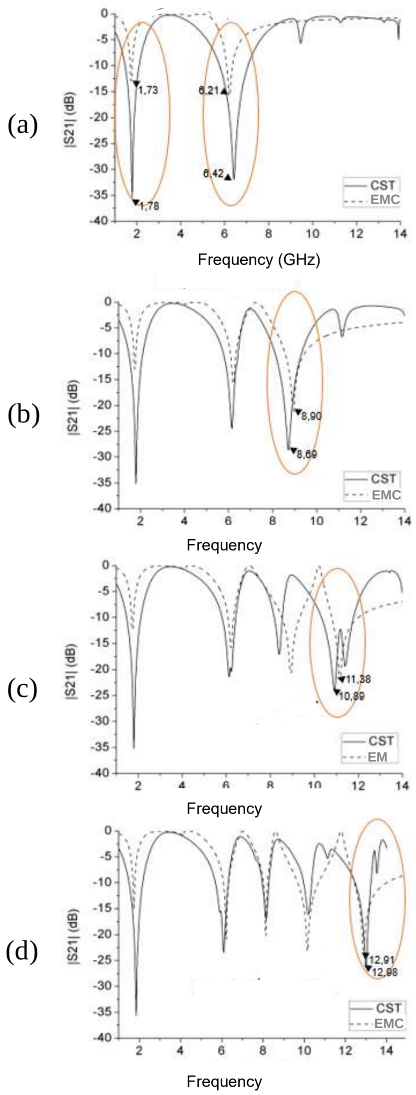


Fig. 7. S21 Comparison of the FSS-JCM iterations using the Software and ECM: (a) Iteration 0; (b) Iteration 1; (c) Iteration 2; (d) Iteration 3.

V. CONCLUSION

An Equivalent Circuit Method (ECM) was successful applied to estimate the transmission coefficient of a modified Jerusalem Cross FSS. This method has the advantage of being a function of only the dimensions and dielectric layer of the FSS, without the needing of a 3D full-wave electromagnetic simulation software. However, this software was used to validate the results obtained by the proposed method. A maximum difference of 6.13% and 3.27% was observed between the ECM and the software for FSS without and with dielectric layer. The ECM used to analyze the FSS was efficient, fast and with sufficient precision to predict the performance of these surfaces.

From the proposed methodology, it is possible to incorporate new procedures into the equivalent circuit model (genetic algorithms, neural networks), to improve the modeling of new devices whose geometries are not common in the literature. And this methodology can be implemented in other open source programming languages such as Python, C ++, and others.

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